

# Dark Energy and Dark Matter interaction: Kernels of volterra type and coincidence problem

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## Abstract

© 2018 by the authors. We study a new exactly solvable model of coupling of the Dark Energy and Dark Matter, in the framework of which the kernel of non-gravitational interaction is presented by the integral Volterra-type operator well-known in the classical theory of fading memory. Exact solutions of this isotropic homogeneous cosmological model were classified with respect to the sign of the discriminant of the cubic characteristic polynomial associated with the key equation of the model. Energy-density scalars of the Dark Energy and Dark Matter, the Hubble function and acceleration parameter are presented explicitly; the scale factor is found in quadratures. Asymptotic analysis of the exact solutions has shown that the Big Rip, Little Rip, Pseudo Rip regimes can be realized with the specific choice of guiding parameters of the model. We show that the Coincidence problem can be solved if we consider the memory effect associated with the interactions in the Dark Sector of the universe.

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## Keywords

Dark Energy, Dark Matter, Memory

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